Pricing Strategy and Coordination in Three-Echelon e-Closed-Loop Supply Chain

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Abstract: In the context of "Internet +", the impact of e-commerce platform services on product market demand should be considered, we investigate a three-echelon E-closed-loop supply chain (E-CLSC). The demand depends on the retail price of products and service level of the e-commerce platform. Base on game theory, we derive the optimal strategies for both the centralized and the decentralized channel scenarios. Subsequently, we analyze the relationship of the optimal strategies and some parameters. We find that: (1) the optimal strategies of the centralized model are better than that of the decentralized model. (2) The optimal strategies and profits of system all increase with the increase of service level's influence on market demand. We finally give a coordination contract to increase social welfare.

1. Introduction

With the development of Internet technology, digital platform economy has become a necessity for everyone. Online mode has brought great convenience, wider market and more profits to both suppliers and consumers, thus, people more tend to spend money on online platforms than offline mode. In addition, with the increasing shortage of natural resources and the improvement of people's awareness of environmental protection, product remanufacturing has been more and more widely recognized [1-3].

At present, many research results have been made in the E-supply chain and closed-loop supply chain, especially operational concepts and operational models. Xiao J studied the cooperation strategy between manufacturer and retailer in E-supply chain [2]. Savaskan et al. analyzed the optimal strategies under different recycling bodies and carried out pareto optimization [3]. Shi et al. addressed the optimal production and the pricing strategies in a CLSC with uncertain demand and return [4]. Ma et al. investigated the optimal pricing decision problem in closed-loop supply chains with marketing effort and fairness concerns [5]. Choi et al. constructed a three-level closed-loop supply chain model and studied the influence of the dominant position of supply chain members on the overall performance of the closed-loop supply chain. As a matter of fact, on the one hand, more and more manufacturers sell products through e-commerce platforms, and on the other hand, they recycle waste products through third-party recyclers. Due to the different operation modes between traditional closed-loop supply chain and E-closed-loop supply chain, it is difficult to apply the research results of traditional closed-loop supply chain directly to E-closed-loop supply chain. Therefore, it is necessary to further study the operation decision of E-closed-loop supply chain.

Based on the above literatures, we construct a three-echelon E-closed-loop supply chain (E-CLSC) model. Subsequently, we derive the optimal strategies for both the decentralized and the centralized channel scenarios. Moreover, we analyze the relationship between the optimal strategies and some parameters. At last, we give a coordination contract to increase social welfare.

2. Problem Descriptions and Model Assumptions

In this paper, we study a three-level CLSC with a manufacturer, an e-commerce platform and a third-party recycler. In the forward supply chain, the manufacturer can either directly use raw materials with the unit cost of production c_m , or recycle used products with the unit cost of remanufacturing c_r to produce a new product and sell it to consumers with a retail price p through the e-commerce platform. The e-commerce platform makes profits from sales commissions to the manufacturer. While in the reverse supply chain, the third-party recycler collects used products with a price a, and the manufacturer takes back used products from the third-party recycler with a transfer prices b. The market demand is a linear function of the retail price and service level

$$D = \phi - p + rs \tag{1}$$

where is the market size, *r* is used to measure the impact of service level on demand. The total service level cost of the retail is expressed as $ks^2/2$, where *k* is the service cost coefficient ^[7]. The used product collection rate τ is introduced to reflect the collection effort and signify the reverse channel performance. In this paper, the collection rate of the third-party recycler can be simplified as follows: $\tau = \sqrt{I/C_L}$, where C_L is a scalar parameter, which is the coefficient of exchange between the collection rate τ and the investment *I*.

There are some assumptions in this paper.

(1) New products and remanufactured products are homogeneous, and the information is symmetric.

(2) Without loss of generality, the behaviour of all agents is favourable, i.e. $\Delta \ge b > a \ge 0$, where $\Delta = c_m - c_r$ is the unit cost savings by remanufacturing a product.

(3) All strategies of E-CLSCs are considered in a single period.

(4) Agents of E-CLSCs are risk-neutral and the e-commerce platform is the market leader.

Based on the above problem formulation and assumptions, decision functions of each agent are given as follows:

$$\pi_{M} = \left(p - \rho - c_{m} + (\Delta - b)\tau\right)\left(\phi - p + rs\right)$$
(2)

$$\pi_N = \rho \left(\phi - p + rs \right) - ks^2/2 \tag{3}$$

$$\pi_T = (b-a)(\phi - p + rs) - C_L \tau^2 \tag{4}$$

3. Decision Models

3.1 Centralized Model

In the centralized model, the manufacturer, the e-commerce platform and the third-party recycler cooperate and make joint decisions. The goal of all agents is to maximize system profits. Thus, the decision function of the system can be expressed as follow:

$$\pi_{SC}^{C} = (p - c_m + (\Delta - a)\tau)(\phi - p + rs) - C_L \tau^2 - ks^2/2$$
(5)

If $k > r^2$, $C_L > (\Delta - a)^2$, the profit function π_{sc}^c is strictly concave in *p*, *s* and τ .

Simultaneous equations $\partial \pi_{sc}^{c} / \partial p = 0$, $\partial \pi_{sc}^{c} / \partial s = 0$ and $\partial \pi_{sc}^{c} / \partial \tau = 0$, we get the proposition 1.

Proposition1 In the centralized model, if $k > r^2$, $C_L > (\Delta - a)^2$, the optimal strategies are given by

$$p^{C^*} = \frac{2C_L c_m (k-r^2) + \phi k (2C_L - (\Delta - a)^2)}{2C_L (2k-r^2) - k (\Delta - a)^2}, s^{C^*} = \frac{2C_L r (\phi - c_m)}{2C_L (2k-r^2) - k (\Delta - a)^2}, \tau^{C^*} = \frac{k (\phi - c_m) (\Delta - a)}{2C_L (2k-r^2) - k (\Delta - a)^2}$$

To ensure $\tau^{C^*} < 1$, we should give the following assumption 1:

the scale parameter
$$C_L > \max\left\{ \left(\Delta - a\right)^2, \frac{k(\Delta - a)^2 + k(\phi - c_m)(\Delta - a)}{2(2k - r^2)} \right\}$$
.

Substituting the value of p^{C^*} , s^{C^*} and τ^{C^*} back in the equations (1) and (5), we get the market demand and the system's profits

$$D^{C^*} = \frac{2C_L k(\phi - c_m)}{2C_L (2k - r^2) - k(\Delta - a)^2}, \pi_{SC}^{C^*} = \frac{C_L k(\phi - c_m)^2}{2C_L (2k - r^2) - k(\Delta - a)^2}$$

3.2 Decentralized Model

As independent economic entities, the manufacturer, the e-commerce platform and the thirdparty recycler make decisions with the goal of maximizing profits. The decision-making sequence is as follows. First, the e-commerce platform determines the unit commission ρ and service level *s*. Second, the manufacturer determines the retailing price *p* and transfer price *b*. Last, the third-party recycler determines the degree of investment recovery *I*, which determines the return rate of used products.

Based on the above analysis, we get the proposition 2-4, which are as follows.

Proposition 2 If $k > r^2$, $C_L > (\Delta - a)^2$, in decentralized model, the optimal strategies of the third-party recycler is given by

$$\tau^{D^*} = \frac{k(\phi - c_m)(\Delta - a)}{2(2C_L(4k - r^2) - k(\Delta - a)^2)}.$$

Proposition 3 If $k > r^2$, $C_L > (\Delta - a)^2$, in decentralized model, the optimal strategies of the manufacturer is given by

$$b^{D^*} = \frac{\Delta + a}{2}, p^{D^*} = \frac{2C_L c_m (k - r^2) + k\phi (6C_L - (\Delta - a)^2)}{2C_L (4k - r^2) - k (\Delta - a)^2}.$$

Proposition 4 If $k > r^2$, $C_L > (\Delta - a)^2$, in decentralized model, the optimal strategies of the ecommerce platform are given by

$$\rho^{D^*} = \frac{k(\phi - c_m) \left(8C_L - (\Delta - a)^2 \right)}{2 \left(2C_L \left(4k - r^2 \right) - k \left(\Delta - a \right)^2 \right)}, \ s^{D^*} = \frac{2C_L r(\phi - c_m)}{2C_L \left(4k - r^2 \right) - k \left(\Delta - a \right)^2}$$

Substituting the value of p^{D^*} , b^{D^*} , ρ^{D^*} , s^{D^*} and τ^{D^*} back in the equations (1)-(5), we get the market demand, the manufacturer's profit, the e-commerce platform's profit and the third party recycler's profit

$$D^{D^*} = \frac{2C_L k (\phi - c_m)}{2C_L (4k - r^2) - k (\Delta - a)^2}, \pi_M^{D^*} = \frac{C_L k^2 (\phi - c_m)^2 (8C_L - (\Delta - a)^2)}{2 (2C_L (4k - r^2) - k (\Delta - a)^2)^2}$$
$$\pi_N^{D^*} = \frac{C_L k (\phi - c_m)^2}{2C_L (4k - r^2) - k (\Delta - a)^2}, \pi_T^{D^*} = \frac{C_L k^2 (\phi - c_m)^2 (\Delta - a)^2}{4 (2C_L (4k - r^2) - k (\Delta - a)^2)^2}$$
$$\pi_{SC}^{D^*} = \frac{C_L k (\phi - c_m)^2 (8C_L (6k - r^2) - 5k (\Delta - a)^2)}{4 (2C_L (4k - r^2) - k (\Delta - a)^2)^2}.$$

4. Strategies Comparison and Analysis

In this section, we compare our results between decentralized and the centralized model. Based on the results summarized in proposition 1-4, Proposition 5 and other interesting observations can be found.

Proposition 5 (1) $p^{C^*} < p^{D^*}$, (2) $s^{C^*} > s^{D^*}$, (3) $\tau^{C^*} > \tau^{D^*}$, (4) $D^{C^*} > D^{D^*}$, (5) $\pi_{SC}^{C^*} > \pi_{SC}^{D^*}$.

Proof Beacause $k > r^2$, $\phi > c_m$, $C_L > (\Delta - a)^2$, thus $C_L (2k - r^2) - k (\Delta - a)^2 > 0$, it is easy to verify that

$$p^{C^{*}} - p^{D^{*}} = -\frac{8C_{L}^{2}k(k-r^{2})(\phi-c_{m})}{\left(2C_{L}(2k-r^{2})-k(\Delta-a)^{2}\right)\left(2C_{L}(4k-r^{2})-k(\Delta-a)^{2}\right)} < 0$$

$$s^{C^{*}} - s^{D^{*}} = \frac{8C_{L}^{2}kr(\phi-c_{m})}{\left(2C_{L}(4k-r^{2})-k(\Delta-a)^{2}\right)\left(2C_{L}(2k-r^{2})-k(\Delta-a)^{2}\right)} > 0$$

$$\tau^{C^{*}} - \tau^{D^{*}} = \frac{k\left(2C_{L}(6k-r^{2})-k(\Delta-a)^{2}\right)\left(\phi-c_{m}\right)(\Delta-a)}{2\left(2C_{L}(2k-r^{2})-k(\Delta-a)^{2}\right)\left(2C_{L}(4k-r^{2})-k(\Delta-a)^{2}\right)} > 0$$

$$D^{C^{*}} - D^{D^{*}} = \frac{8C_{L}^{2}k^{2}(\phi-c_{m})}{\left(2C_{L}(2k-r^{2})-k(\Delta-a)^{2}\right)\left(2C_{L}(4k-r^{2})-k(\Delta-a)^{2}\right)} > 0$$

$$\pi^{C^{*}}_{SC} - \pi^{D^{*}}_{SC} = \frac{C_{L}k(\phi-c_{m})^{2}\left(4\left(2C_{L}(4k-r^{2})-k(\Delta-a)^{2}\right)^{2}-\left(2C_{L}(2k-r^{2})-k(\Delta-a)^{2}\right)\left(8C_{L}(6k-r^{2})-5k(\Delta-a)^{2}\right)\right)}{4\left(2C_{L}(2k-r^{2})-k(\Delta-a)^{2}\right)\left(2C_{L}(4k-r^{2})-k(\Delta-a)^{2}\right)^{2}}$$

Let

$$J(k) = 4\left(2C_{L}\left(4k - r^{2}\right) - kG\right)^{2} - \left(2C_{L}\left(2k - r^{2}\right) - kG\right)\left(8C_{L}\left(6k - r^{2}\right) - 5kG\right)$$

if $k = r^2$, we know $J(r^2) = r^4 \left(64C_L^2 + \left(2C_L - (\Delta - a)^2 \right) (\Delta - a)^2 \right) > 0$. Taking derivative of J(k) with respect to k, we get $\partial J(k) / \partial k = 128k \left(C_L + (\Delta - a)^2 \right) \left(C_L - (\Delta - a)^2 \right) + 2C_L \left(4k - r^2 \right) (\Delta - a)^2 > 0$, thus, if $k > r^2$, J(k) > 0. That's to say $\pi_{SC}^{C*} - \pi_{SC}^{D*} > 0$.

Proposition 5 confirms that the centralized mode is better than decentralized mode. That's to say that the retail price of centralized mode is lower that of decentralized mode, the service level, return rate and profits of the system are higher in the centralized mode.

5. Coordination Mechanism Design

From the above analysis, we know that the e-commerce platform is a leader, the manufacturer is a follower and the third party recycler is the last mover in the three-level E-CLSC. Thus, we need to provide a two-part tariff contract to coordinate them echelon by echelon.

Because the e-commerce platform is the forward channel leader and the manufacturer is the reverse channel leader. First, the e-commerce platform should coordinate the forward supply chain. Next, the manufacturer coordinates the reverse supply chain. Generally speaking, if the supply chain system achieves coordination, the retail price, service level and the collection rate under the decentralized scenarios should equal that under the centralized scenarios. According to proposition 5, we know that $p^{C^*} < p^{D^*}$, $s^{C^*} > s^{D^*}$ and $\tau^{C^*} > \tau^{D^*}$. Thus, the problem of the e-commerce platform is given as follows:

$$\max_{\rho, F_1} \pi_N = \rho \left(\phi - p(\rho, s) + rs \right) - \frac{k}{2} s^2 + F_1$$

s.t.
$$\begin{cases} p \in \arg \max_{p,b} \left\{ \left(p - c_m - \rho + (\Delta - b) \tau \right) (\phi - p + rs) - F_1 \right\} \\ s = s^{C^*}, \pi_M \ge \pi_M^{D^*}, \pi_N \ge \pi_N^{D^*} \end{cases}$$

where F_1 is a fixed payment, paid by the manufacturer to the e-commerce platform. $s = s^{C^*}$ means that the e-commerce platform makes a concession to secure the contract, $\pi_M \ge \pi_M^{D^*}$ and $\pi_N \ge \pi_N^{D^*}$ mean that the coordinated decision model is better than the decentralized decision model.

On the other hand, the problem of the manufacturer is given as follows:

$$\max_{b,F_{2}} \pi_{M} = (p - c_{m} + (\Delta - b)\tau)(\phi - p + rs^{C^{*}}) + F_{2}$$

s.t.
$$\begin{cases} \tau \in \arg\max_{p,b} \left\{ (b - a)\tau(\phi - p + rs) - C_{L}\tau^{2} - F_{2} \right\} \\ p = p^{C^{*}}, s = s^{C^{*}}, \pi_{M} \ge \pi_{M}^{D^{*}}, \pi_{T} \ge \pi_{T}^{D^{*}} \end{cases}$$

where F_2 also is a fixed payment, paid by the third party recycler to the manufacturer. $p = p^{C^*}$ and $s = s^{C^*}$ mean that the manufacturer and the e-commerce platform both make a concession to secure the contract, $\pi_M \ge \pi_M^{D^*}$ and $\pi_T \ge \pi_T^{D^*}$ mean that the coordinated decision model is better than the decentralized decision model.

By solving the above model, we can get the proposition 6.

In the coordinated model, the optimal strategies of the manufacturer are given by

$$p^{*} = p^{C*} = \frac{2C_{L}c_{m}(k-r^{2}) + \phi k \left(2C_{L} - (\Delta - a)^{2}\right)}{2C_{L}(2k-r^{2}) - k \left(\Delta - a\right)^{2}}, s^{*} = s^{C*} = \frac{2C_{L}r(\phi - c_{m})}{2C_{L}(2k-r^{2}) - k \left(\Delta - a\right)^{2}}, r^{*} = \tau^{C*} = \frac{k(\phi - c_{m})(\Delta - a)}{2C_{L}(2k-r^{2}) - k \left(\Delta - a\right)^{2}}, F^{*}_{1} = \pi^{D*}_{M}(\rho, s^{C*}, \tau^{C*}) - \pi^{D*}_{M}, F^{*}_{2} = \pi^{D*}_{T}(b, p^{*}, s^{C*}) - \pi^{D*}_{T}$$

6. Numerical Simulation

Next, we will take numerical studies to further examine the effect of *r* on the optimal strategies, and assume that the effect of service level on product demand is less than that of sales price on product demand, i.e. $r \in (0,1)$. Let $\phi = 300$, $C_L = 3000$, $c_m = 50$, $\Delta = 35$, k = 10, a = 5. Figure 1-4 show the relationship between the optimal strategies and parameter *r* and validate the proposition 5.



Figure 1 The Relationship between the Optimal Retail Price and Parameter r



Figure 2 The Relationship between the Optimal Service Level and Parameter r

Figure 1 shows that the optimal retail price of centralized mode is lower than that of decentralized mode and the optimal prices are monotonically increasing in the parameter r. Figure 2

shows that the optimal service level of centralized mode is higher than that of decentralized mode and the optimal service levels are also monotonically increasing in the parameter r. Furthermore, the gap is widening along as r increases between centralized mode and decentralized mode. Figure 3 shows that the optimal collection rate of centralized mode is higher than that of decentralized mode and the optimal collection rates are also monotonically increasing in the parameter r. Figure 4 shows that the optimal system's profits of centralized mode is higher than that of decentralized mode and the optimal system's profits are also monotonically increasing in the parameter r.



Figure 3 The Relationship between the Optimal Collection Rate and Parameter r



Figure 4 The Relationship between the Optimal Profits of e-Clsc and Parameter r

7. Conclusion

In this paper, we address the optimal strategies of a three-echelon E-CLSC. Base on game theory, the optimal strategies are derived for both the decentralized and the centralized models. By comparing the results obtained, we find that the centralized decision mode is superior to the decentralized decision mode, and the optimal strategies increase monotonically with the increase of elasticity coefficient of service level.

We only consider a single e-commerce platform in this paper, but in reality there are often multiple e-commerce platforms. Next, we shall consider the impact of e-commerce platform competition on the optimal strategies in the future.

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